PENGUIN SERIES

Accurate Mathematics

A Coursebook in Mathematics with Activities

Written by :
R.D. Verma

6

PETERSON PRESS
EDUCATION LEADS
A UNIT OF
NAMAN PUBLISHING (INDIA) PVT. LTD.

Accurate Mathematics

A Coursebook in Mathematics with Activities

Typesetting & Graphics by

Dreamshapers # 9897035745

Editors
Damini Kaushik
Yatin Kumar

Published by

PETERSON PRESS
EDUCATION LEADS
A UNIT OF
NAMAN PUBLISHING (INDIA) PVT. LTD.

Redg. Office
Behind Silver Line School, Laxmipuram,
Rajpur Chungi, AGRA-282001
Mobile : +91-9837004559, 983719441
Telefax : 0562-2481926
e-mail : namanpublishing@yahoo.com

Preface

Accurate Mathematics is an innovative series for classes 1 to 8 which is specially designed for the children of new generation. Children should enjoy learning Mathematics rather than be afraid of it. They should pose and solve meaningful problems with ease. The contents of the books are complete and carefully graded as per the novel approach to the teaching of 'Mathematics hands on experience', in perfect co-ordination with resources available in the learner's immediate environment. The series follows the 'explain, comprehend and practise essential drill application' approach. The chapters provide a clear understanding, emphasize an investigative and exploratory approach to teaching. Wherever necessary, theory is presented precisely in a style tailored to act as a tool for teachers and students.

The theory is presented in a very simple language and supported with examples from everyday life.

A large number of objective questions have been included, which will help students quickly test their knowledge and skill.

A separate chapter titled as 'Activities' has been included to connect maths with real-life situations.

Test papers will help learners practise and apply the concepts learnt.

Every effort has been made to keep the series error-free. However, constructive suggestions for the improvement of the next edition will be highly appreciated.

– Publisher

Contents

1 Number System

Numbers form an integral part of our life. In our earlier classes, we have dealt with counting numbers 1, 2, 3, …… etc. These counting numbers are called natural numbers. Counting numbers have no end. They go on forever.

We may express numbers in figures as well as in words.

NOTATION : *Writing a number in figures is called notation.*

NUMERATION : *Writing a number in words is called numeration.*

HINDU-ARABIC SYSTEM OF WRITING NUMBERS

In the Hindu-Arabic system, we use ten symbols, namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9, called **digits** or **figures,** to represent any number.

A group of digits, denoting a number, is called a **numeral.**

We use place-value system to represent a number.

For a given numeral, we start from the extreme right as : Ones, Tens, Hundreds, Thousands, Ten thousands, Lakhs, Ten lakhs, etc.

Example 1. *Given below are some numbers arranged in a place-value table. Write each number in words and put it in the expanded form.*

Period ⟶	Lakhs		Thousands		Ones		
	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
(a)				8	5	2	7
(b)			7	1	6	5	4
(c)		2	4	5	7	0	3
(d)	4	8	6	2	3	7	1

Solution: (a) The given numeral is 8527.

It is '*Eight thousand five hundred twenty-seven*'.

In expanded form, we write it as:

$8527 = (8 \times 1000) + (5 \times 100) + (2 \times 10) + (7 \times 1)$

(b) The given numeral is 71654.

It is '*Seventy-one thousand six hundred fifty-four*.'

In expanded form, we write it as:

$71654 = (7 \times 10000) + (1 \times 1000) + (6 \times 100) + (5 \times 10) + (4 \times 1)$

(c) The given numeral is 245703.

It is '*Two lakh forty-five thousand seven hundred and three*'.

In expanded form, we write it as:

$245703 = (2 \times 100000) + (4 \times 10000) + (5 \times 1000) + (7 \times 100) + (0 \times 10) + (3 \times 1)$

(d) The given numeral is 4862371.

It is '*Forty-eight lakh sixty-two thousand three hundred seventy-one.*'

In expanded form, we write it as:

$4862371 = (4 \times 1000000) + (8 \times 100000) + (6 \times 10000) + (2 \times 1000) + (3 \times 100) + (7 \times 10) + (1 \times 1)$

NUMBER SYSTEM EXTENDED FURTHER

We know that the largest 7-digit number is **9999999.**

It is *ninety-nine lakh ninety-nine thousand nine hundred ninety-nine.*

On adding 1 to it, we get

$(9999999 + 1)$ = **10000000,** called **one crore.**

Thus, the smallest 8-digit number is one crore.

The largest 8-digit number is **99999999.**

It is *nine crore ninety-nine lakh ninety-nine thousand nine hundred ninety-nine.*

Thus, we extend our place-value chart to have

One, Tens, Hundreds, Thousands, Ten thousands, Lakhs, Ten lakhs, Crores and Ten crores, etc.

Example 2. *Given below are the numerals*

(a) 64513807 and (b) 742161058.

Put them in the place-value chart. Also write them in words.

Put each one of them in the expanded form.

Solution: The new place-value chart is given below :

Period ⟶	Crores		Lakhs		Thousands		Ones		
	Ten crores	Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
(a)		6	4	5	1	3	8	0	7
(b)	7	4	2	1	6	1	0	5	8

(a) The given numeral is 64513807

It is *six crore forty-five lakh thirteen thousand eight hundred and seven.*

Its expanded form is :

$64513807 = (6 \times 10000000) + (4 \times 1000000) + (5 \times 100000) + (1 \times 10000) + (3 \times 1000) + (8 \times 100) + (0 \times 10) + (7 \times 1)$

(b) The given numeral is 742161058.

It is *seventy-four crore twenty-one lakh sixty-one thousand fifty-eight.*

Its expanded form is :

$742161058 = (7 \times 100000000) + (4 \times 10000000) + (2 \times 1000000) + (1 \times 100000) + (6 \times 10000) + (1 \times 1000) + (0 \times 100) + (5 \times 10) + (8 \times 1)$

PERIODS: *Crores, Lakhs, Thousands and Hundreds are known as periods.*

SEPARATING PERIODS IN HINDU-ARABIC SYSTEM

The various periods are :

(Crores), (Lakhs), (Thousands), (Hundreds + Tens + Ones)

We insert comma after each period.

Example 1. *Write* 64513807*, separating periods.*

Solution: Separating periods, we have

Crores	Lakhs	Thousands	Ones		
			H	T	O
6	45	13	8	0	7

Using commas, we write it as 6,45,13,807.

Example 2. *Write* 742161058, *separating periods.*

Solution: Separating periods, we have

Crores	Lakhs	Thousands	Ones		
			H	T	O
74	21	61	0	5	8

Using commas, we write it as 74,21,61,058.

FACE VALUE OF A DIGIT IN A NUMERAL

The face value of a digit remains as it is, whatever place it may be occupying in the place-value chart.
Thus, the face value of 5 is always 5, wherever it may be.

PLACE VALUE OF A DIGIT IN A NUMERAL

The place value of a digit in a numeral depends upon the place it occupies in the place-value chart.
If 6 occurs at ones place, its place value = 6 ones = $(6 \times 1) = 6$.
If 6 occurs at tens place, its place value = 6 tens = $(6 \times 10) = 60$.
If 6 occurs at hundreds place, its place value = 6 hundreds = $(6 \times 100) = 600$, and so on.
Some more examples are given below.

Example: *Find the place value of each digit in* '61823705'.

Solution: In 61823705, we have

place value of 5 = 5 ones = $(5 \times 1) = 5$;
place value of 0 = 0 tens = $(0 \times 10) = 0$;
place value of 7 = 7 hundreds = $(7 \times 100) = 700$;
place value of 3 = 3 thousands = $(3 \times 1000) = 3000$;
place value of 2 = 2 ten thousands = $(2 \times 10000) = 20000$;
place value of 8 = 8 lakhs = $(8 \times 100000) = 800000$;
place value of 1 = 1 ten lakh = $(1 \times 1000000) = 1000000$;
place value of 6 = 6 crores = $(6 \times 10000000) = 60000000$.

Solved Examples

Example 1. *Separate the periods of the numeral* '82463751' *by commas and write it in words.*

Solution: Separating periods, we have

Crores	Lakhs	Thousands	Ones		
			H	T	O
8	24	63	7	5	1

Using commas, we write it as 8,24,63,751.
In words, we write it as:
'eight crore twenty-four lakh sixty-three thousand seven hundred fifty-one'.

Example 2. *Find the difference of the place values of the two* 9*s in* 95810964.

Solution: The place value of 9 at crores place 9 crores $= (9 \times 10000000) = 90000000$

The place value of 9 at hundreds place $= 9$ hundreds $= (9 \times 100) = 900$
Required difference $= (90000000 - 900) = 89999100$

Example 3. *How many* 8*-digit numbers are there in all*?

Solution: The largest 8-digit number $= 99999999$

The smallest 8-digit number $= 10000000$
Number of all 8-digit numbers $= (99999999 - 10000000) + 1$
$= (89999999 + 1) = 90000000$
$=$ nine crores
Hence, there are in all nine crores of 8-digit numbers.

Example 4. *How many* 5*-digit numbers are there in all*?

Solution: The largest 5-digit number $= 99999$

The smallest 5-digit number $= 10000$
Number of all 8-digit numbers $= (99999 - 10000) + 1$
$= (89999 + 1) = 90000$
Hence, the number of all 5-digit numbers is ninety thousand.

Example 5. *Write all* 3*-digit numbers using the digits* 1, 3, 5, *taking each digit only once.*

Solution: Keeping 1 at the ones place, the numbers formed are 351 and 531.

Keeping 3 at the ones place, the numbers formed are 153 and 513.
Keeping 5 at the ones place, the numbers formed are 135 and 315.
So, the required numbers are 351, 531, 153, 513, 135 and 315.

Example 6. *Write the smallest* 8*-digit number having four different digits.*

Solution: Four smallest digits are 0, 1, 2, 3.

Hence, the required number is 10000023.

INTERNATIONAL SYSTEM OF NUMERATION

In the international system of numeration adopted by all the countries throughout the world, the place-value chart is as follows.

Period ⟶	**Billions**		**Millions**			**Thousands**			**Ones**		
	Ten billions	Billions	Hundred millions	Ten millions	Millions	Ten Thousands	Hundred thousands	Thousands	Hundreds	Tens	Ones
				4	5	2	3	6	7	9	0
			TC	**C**	**TL**	**L**	**T Th**	**Th**	**H**	**T**	**O**

The number '45236790' the international system will be read as :
'forty-five' million two hundred and thirty-six thousand seven hundred ninety.

It is clear from the table that the above number in Hindu-Arabic system is given by *'four crore fifty-two lakh thirty-six thousand seven hundred ninety'*.

Example. *Rewrite each of the following numbers with proper commas, using international place-value chart :*

(a) 64795 (b) 2397586 (c) 38526471

Also, write the number name of each in the international system.

Solution: Let us arrange the given numerals in an international place-value chart. Then separating the periods, we write them as shown below.

	Millions			Thousands			Ones			
	HM	TM	M	H Th	T Th	Th	H	T	0	
(a)					6	4	7	9	5	64,795
(b)			2	3	9	7	5	8	6	2,397,586
(c)		3	8	5	2	6	4	7	1	38,526,471
		C	TL	L	TTh	Th	H	T	P	

In the international system, we write them in words as :

(a) Sixty-four thousand seven hundred ninety-five

(b) Two million three hundred ninety-seven thousand five hundred eighty-six

(c) Thirty-eight million five hundred twenty-six thousand four hundred seventy-one.

In the Hindu-Arabic system, these numbers are :

(a) Sixty-four thousand seven hundred ninety-five

(b) Twenty-three lakh ninety-seven thousand five hundred eighty-six

(c) Three crore eighty-five lakh twenty-six thousand four hundred seventy-one.

Exercise 1A

1. Write the numeral for each of the following numbers :

(a) Eight thousand fourteen

(b) Forty-three thousand sixty-two

(c) Two lakh three thousand six hundred five

(d) Fifty lakh twenty thousand seven

(e) Seven crore four lakh thirty-seven

(f) Six crore five lakh four thousand three hundred three

(g) Fifteen crore fifty lakh twenty thousand sixty-eight

(h) Twelve crore twelve lakh twenty thousand twelve

2. Write each of the following numbers in words :

(a) 74,006 (b) 8,08,086 (c) 43,30,028

(d) 2,06,08,013 (e) 6,20,04,705 (f) 7,29,06,009

(g) 28,08,08,800 (h) 7,26,40,908 (i) 5,50,50,050

3. Write each of the following numbers in expanded form :

(a) 26,657 (b) 4,09,816 (c) 13,04,508

(d) 4,25,27,382 (e) 8,08,08,008 (f) 8,20,30,620

4. Write the corresponding numeral for each of the following :
   (a) $5\times10000+3\times1000+4\times100+7\times10+5\times1$
   (b) $4\times100000+6\times10000+2\times1000+3\times100+5\times10+2\times1$
   (c) $3\times10000000+2\times100000+5\times1000+8\times100+6\times1$
   (d) $8\times1000000+3\times100000+2\times1000+9\times100+3\times1$
5. Find the difference between the place value and the face value of 7 in 27650934.
6. Find the difference between the place value and the two nines in 79520986.
7. How many 7-digit numbers are there in all?
8. How many 6-digit numbers are there in all?
9. How many thousands make a crore?
10. How many thousands make a lakh?
11. What comes just before 10000000?
12. What comes just before 9900000?
13. What comes just after 9547999?
14. Find the difference between the number 738 and that obtained on reversing its digits.
15. Write the largest number of different digits formed by using the digits 2, 4, 0, 3, 6 and 9.
16. Write all 3-digit numbers using 2, 3, 4 taking each digit only once.
17. Write the smallest number of different digits formed by using the digits 3, 1, 0, 5 and 7.
18. Write each of the following in figures in the international place-value chart:
   (a) Thirty million one hundred five thousand sixty-three
   (b) Fifty-two million two hundred five thousand six
   (c) Five million five thousand five
19. Rewrite each of the following numerals with proper commas, using the international place-value chart. Also, write the number name of each in the international system.
   (a) 735821 (b) 6057894 (c) 56943821
   (d) 37502093 (e) 89350064 (f) 90703006

COMPARISON OF NUMBERS

While comparing two numbers, we have to follow the following rules :

Rule 1. The number with less digits is less than the number with more digits.

Rule 2. Suppose we have to compare two numbers having the same number of digits, then we proceed as under.

Step 1. First compare the digits at the leftmost place in both the numbers.

Step 2. If they are equal in value then compare the second digits from the left.

Step 3. If the second digits from the left are equal then compare the third digits from the left.

Step 4. Continue until you come across unequal digits at the corresponding places. Clearly, the number with greater such digit is the greater of the two.

The following examples will make the ideas more clear.

Solved Examples

Example 1. *Compare* 8739608 *and* 10329643?

Solution: 8739608 has 7 digits while 10329643 has 8 digits.

We know that a number with more digits is greater.

$\therefore$ $10329643 > 8739608$

Example 2. *Which is greater :* 85740263 *or* 85610784?

Solution: Let us arrange the given numbers in a place-value chart, as shown below.

Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
8	5	7	4	0	2	6	3
8	5	6	1	0	7	8	4

Clearly, both the numbers have 8 digits.

At the crores place both have the same digit, namely, 8.

At the ten lakhs place both have the same digit, namely, 5.

But, at the lakhs place, the first number has 7 while the second has 6.

Clearly, $7 > 6$

$85740263 > 85610784$

Example 3. *Arrange the following numbers in ascending order :*

4874325, 29450328, 895782, 4890534, 29542067

Solution: Let us arrange the given numbers in a place-value chart, as shown below.

Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
	4	8	7	4	3	2	5
2	9	4	5	0	3	2	8
		8	9	5	7	8	2
	4	8	9	0	5	3	4
2	9	5	4	2	0	6	7

Out of the given numbers one is a 6-digit number, two are 7-digit numbers and two are 8-digit numbers.

6-digit number is 895782.

In 7-digit numbers, we have $4874325 < 4890534$

In 8-digit numbers, we have $29450328 < 29542067$

Hence, the given numbers in ascending order are:

$895782 < 4874325 < 4890534 < 29450328 < 29542067$

Example 4. *Arrange the following numbers in descending order :*

63872604, 4965328, 63890503, 5023145, 576943

Solution:

Ten Crores	Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
	6	3	8	7	2	6	0	4
		4	9	6	5	3	2	8
	6	3	8	9	0	5	0	3
		5	0	2	3	1	4	5
			5	7	6	9	4	3

In 8-digit numbers, clearly 63890503 > 63872604 (9 > 7)
In 7-digit numbers, clearly 5023145 > 4965328 (5 > 4)
In 6-digit numbers is 576943, which is clearly the smallest.
∴ 63890503 > 63872604 > 5023145 > 4965328 > 576943
Hence, the given numbers in descending order are :
63890503, 63872604, 5023145, 4965328, 576943

Exercise 1B

1. Fill in each of the following boxes with the correct symbol > or < :
(a) 2004578 ☐ 898976 (b) 4683025 ☐ 20346502
(c) 4365890 ☐ 4370263 (d) 20468790 ☐ 22354678
(e) 38697492 ☐ 8976523 (f) 58994602 ☐ 58995032

2. Write each of the following numbers in ascending order :
(a) 9873426, 24615019, 990357, 9874012, 24620010
(b) 56943201, 5694437, 56944000, 5695440, 56943300
(c) 700087, 8014257, 8015032, 10012458, 8014306
(d) 1020304, 893245, 980134, 1021403, 893425, 1020216

3. Write each of the following numbers in descending order :
(a) 63521047, 7354206, 63514759, 7355014, 102345680
(b) 5032786, 23794206, 5032790, 23756819, 987876
(c) 190909, 1808088, 16060666, 16007777, 181888, 1808090
(d) 199988, 1704382, 200175, 1702497, 201200, 1712040

WORD PROBLEMS ON NUMBER OPERATIONS

Illustrative Examples

Example 1. *Narayan earned* ₹ 3648970 *in* 2018. *Next year, his earning was increased by* ₹ 956880. *What was his earning in* 2019?

Solution: Earning of Narayan in 2018 = ₹ 3648970
Increase in earning during next year = ₹ 956880

Narayan's earning in 2019 = ₹ (3648970 + 956880) = ₹ 4605850

	1	1	1	1	1	1	
	TL	L	T Th	Th	H	T	O
	3	6	4	8	9	7	0
+		9	5	6	8	8	0
	4	6	0	5	8	5	0

Hence, Narayan's earning in 2019 is ₹ 4605850

Example 2. *The difference between two numbers is* 9476583. *If the smaller number is* 6873547, *find the greater number.*

Solution: Difference between the two numbers = 9476583

Smaller number = 6873547

Hence, greater number = (9476583 + 6873547)

	1	1	1	1	1	1	1	
	C	TL	L	T Th	Th	H	T	O
		9	4	7	6	5	8	3
+		6	8	7	3	5	4	7
	1	6	3	5	0	1	3	0

Hence, the greater number is 16350130.

Example 3. *The population of a city in* 2017 *was* 14693675. *In* 2018, *the population became* 18002403. *Find the increase in the population.*

Solution: Population in the year 2018 = 18002403

Population in the year 2017 = 14693675

Increase in population = (18002403 – 14693675)

	C	TL	L	T Th	Th	H	T	O
	1	8	0	0	2	4	0	3
–	1	4	6	9	3	6	7	5
		3	3	0	8	7	2	8

Hence, the increase in population is 3308728.

Example 4. *The cost of a TV is* ₹ 22875. *What is the cost of* 465 *such TVs*?

Solution: Cost of 1 TV = ₹ 22875

Cost of TVs = ₹ (22875 × 465)

= ₹ 10636875

Hence, the cost of 465 TVs is ₹ 10636875

		2	2	8	7	5	
			×	4	6	5	
	1	1	4	3	7	5	
1	3	7	2	5	0	×	
9	1	5	0	0	×	×	
1 0	6	3	6	8	7	5	

Example 5. *The mass of each gas cylinder is* 15 *kg* 450 *g. What is the total mass of* 14 *such cylinders?*

Solution: Mass of 1 cylinder = 15 kg 450 g

Mass of 18 cylinders = (15 kg 450 g) × 14

= 216 kg 300 g

Hence, the total mass of 14 cylinders is 216 kg 300 g.

(+6)	
kg	g
15	450
	× 14
216	300

Example 6. *The piece of cloth required for a kurta is* 3 *m* 15 *cm. How much cloth will be required for* 18 *such kurtas?*

Solution: Cloth required for 1 kurta = 3 m 15 cm

Cloth required for 18 kurtas = (3 m 15 cm) × 18

= 56 m 70 cm.

Hence, the cloth required for 18 kurtas will be 56 m 70 cm long.

(+2)	
m	cm
3	15
	× 18
56	70

Example 7. *The cost of 16 houses constructed by Subh Luxmi Awas Pariyojna is* ₹ 24809520. *Find the cost of each house.*

Solution: Total cost of 16 houses = ₹ 24809520

Cost of each house = ₹ (24809520) ÷ 16

= ₹ 1550595.

Hence, the cost of each house is ₹ 1550595.

```
16)24809520(1550595
 - 16
   88
 - 80
    80
  - 80
     95
   - 80
     152
   - 144
       80
     - 80
       ×
```

Example 8. *A bus covers* 1002 *km in* 16 *hours. At what speed per hour does the bus move?*

Solution: Distance covered in 16 hours = 1002 km

Distance covered in 1 hour = (1002 km) ÷ 16

= 62 km 625 m

Hence, the speed of the bus is 62 km 625 m per hour

```
16)1002 km(62 km
  - 96
    42
   - 32
     10 km
 × 1000
16)10000 m(625 km
  - 96
    40
   - 32
     80
   - 80
     ×
```

Exercise 1C

1. The number of persons who visited the holy shrine of Kedarnath Dham during last two consecutive years was 13789509 and 12976498 respectively. How many persons visited the shrine during these two years?
2. Last year, three rice mills produced 24809565 bags, 18738576 bags and 9564568 bags of rice respectively. How many bags were produced by all the three factories during last year?
3. A number exceeds 37684955 by 3615045. What is that number?

4. There were three candidates in an election. They received 687905 votes, 495086 votes and 93756 votes respectively. The number of invalid votes was 13849. If 25467 persons did not vote, find how many votes were registered.
5. In a particular year, a company produced 8765435 bulbs. Next year, the number of bulbs produced was 1378689 more than those produced in the preceding year.
   (a) How many bulbs were produced during the second year?
   (b) How many bulbs were produced during these two years?
6. The sale receipt of a company during a year was ₹ 20956480. Next year, it increased by ₹ 6709570. What was the total sale receipt of the company during these two years?
7. The total population of a town is 28756304. If the number of females is 16987059, find the number of males in the town.
8. By how much is 5643879 smaller than one crore?
9. What number must be subtracted from 11010101 to get 2635967?
10. The sum of two numbers is 10750308. If one of them is 8967519, what is the other number?
11. By how much is 13246510 larger than 4658642?
12. Rajat had ₹ 20000000 with him. He spent ₹ 13607085 on buying a farm. How much money is left with him?
13. Ramesh had ₹ 10672540 with him. He gave ₹ 4836980 to his wife, ₹ 3964790 to his son and the rest to his daughter. How much money was received by the daughter?
14. The cost of table is ₹ 1525. How much will 525 such tables cost?
15. A factory produces 6985 pens per day. How many pens will it produce in 343 days?
16. Mr Gupta saves ₹ 7645 every month. How much money will he save in 15 years?
17. An helicopter covers 1275 km in 1 hour. How much distance will it cover in 52 hours?
18. The product of two numbers is 13421408. If one of the numbers is 364, find the other.
19. If 36 flats cost 68251500, what is the cost of each such flat?
20. The mass of a cylinder filled with gas is 32 kg 650 g and the mass of the empty cylinder is 15 kg 280 g. How much is the mass of the gas contained in it?
21. In order to make a kurta, a length of 3 m 75 cm of cloth is needed. How much length of the cloth will be required for 12 such shirts?
22. For making 8 trousers of the same size, 14 m 80 cm of cloth is needed. How much cloth will be required for each such trouser?
23. The mass of a box is 2 kg 750 g. What is the total mass of 14 such boxes?
24. The total mass of 8 packets, each of the same size, is 10 kg 600 g. What is the mass of each such packet?
25. A wire of length 30 m has been divided into 6 pieces of the same length. What is the length of each piece?

ESTIMATION

In our daily life, we come across many situation in which approximate figure is given such as about 30,000 visitors came yesterday to see the Taj Mahal. It does not mean that only 30,000 visitors came to see the Taj Mahal, it may be 29,850 or 30,215, etc.

So, estimation means just an approximate number, not exact number.

There are certain rules for rounding off a number to the nearest tens, hundreds, thousands, etc.

Rounding off a Number to the Nearest Ten

Step 1: See the ones digit of the given number.

Step 2: If ones digit is less than 5, replace ones digit by 0 and keep the other digits as they are.

Step 3: If ones digit is 5 or more, increase tens digit by 1 and replace ones digit by 0.

Rounding off a Number to the Nearest Hundred

Step 1: See the *tens digit* of the given number.

Step 2: If tens digit is less than 5, *replace each one of tens and ones digits by* 0 *and keep the other digits as they are.*

Step 3: If this digit is 5 or more, *increase hundreds digit by* 1 *and replace each digit on its right by* 0.

Rounding off a Number to the Nearest Thousand

Step 1: See the *hundreds digit* of the given number.

Step 2: If hundreds digit is less than 5, replace each one of hundreds, tens and ones digits by 0 and keep the other digits as they are.

Step 3: If hundreds digit is 5 or more, increase thousands digit by 1 and replace each digit on its right by 0.

We may extend the ideas for larger numbers.

Solved Examples

Example 1. *Round off each of the following numbers to the nearest ten :*

(a) 43 (b) 386 (c) 6384 (d) 4325

Solution: (a) In 43, the ones digit is $3 < 5$.

$\therefore$ the required rounded number = 40

(b) In 386, the ones digit is $6 > 5$.

$\therefore$ the required rounded number = 390

(c) In 6384, the ones digit is $4 < 5$.

$\therefore$ the required rounded number = 6380

(d) In 4325, the ones digit is $5 = 5$.

$\therefore$ the required rounded number = 4330

Example 2. *Round off each of the following numbers to the nearest hundred :*

(a) 347 (b) 4558 (c) 24869 (d) 3245

Solution: (a) In 347, the tens digit is $4 < 5$.

$\therefore$ the required rounded number = 300

(b) In 4558, the tens digit is $5 = 5$.

$\therefore$ the required rounded number = 4600

(c) In 24869, the tens digit is $6 > 5$.

$\therefore$ the required rounded number = 24900

(d) In 3245, the tens digit is $4 < 5$.

$\therefore$ the required rounded number = 3200

Example 3. *Round off each of the following numbers to the nearest thousand :*

(a) 1864 (b) 2845 (c) 25370 (d) 32647

Solution: (a) In 1864, the hundreds digit is $8 > 5$.

$\therefore$ the required rounded number = 2000

(b) In 2845, the hundreds digit is $8 > 5$.

$\therefore$ the required rounded number = 3000

(c) In 25370, the hundreds digit is $3 < 5$.

$\therefore$ the required rounded number = 25000

(d) In 32647, the hundreds digit is $6 > 5$.

$\therefore$ the required rounded number = 33000

ESTIMATION

ESTIMATING THE SUMS

Example 1. *Estimate the sum* $(73 + 88)$ *to the nearest ten.*

Solution: 73 estimated to the nearest ten = 70

88 estimated to the nearest ten = 90

Hence, the required estimation = $(70 + 90) = 160$

Example 2. *Estimate the sum* $(357 + 122)$ *to the nearest ten.*

Solution: 357 estimated to the nearest ten = 360

122 estimated to the nearest ten = 120

Hence, the required estimation = $(360 + 120) = 480$

Example 3. *Estimate the sum* $(384 + 232)$ *to the nearest hundred.*

Solution: 384 estimated to the nearest hundred = 400

232 estimated to the nearest hundred = 200

Hence, the required estimation = $(400 + 200) = 600$

Example 4. *Estimate the sum* $(21397 + 27807 + 42505)$ *to the nearest thousand.*

Solution: 21397 estimated to the nearest thousand = 21000

27807 estimated to the nearest thousand = 28000

42505 estimated to the nearest thousand = 43000

Hence, the required estimation = $(21000 + 28000 + 43000) = 92000$

Example 5. *Estimate the difference* $(782 - 367)$ *to the nearest hundred.*

Solution: 782 estimated to the nearest hundred = 800

367 estimated to the nearest hundred = 400

Hence, the required estimation = $(800 - 400) = 400$

Exercise 1D

1. Round off each of the following numbers to the nearest ten :
(a) 27 (b) 185 (c) 2778 (d) 27489

2. Round off each of the following numbers to the nearest hundred :
(a) 924 (b) 2158 (c) 54237 (d) 89376

3. Round off each of the following numbers to the nearest thousand :
(a) 875 (b) 5937 (c) 27826 (d) 37473

4. Round off each of the following numbers to the nearest ten thousand :
(a) 28321 (b) 47423 (c) 23680 (d) 381529

5. Estimate each sum to the nearest ten :
(a) $(46 + 23)$ (b) $(54 + 87)$ (c) $(12 + 58)$
(d) $(538 + 276)$ (e) $(356 + 275)$ (f) $(463 + 182)$

6. Estimate each sum to the nearest hundred :
(a) $(170 + 395)$ (b) $(236 + 689)$ (c) $(458 + 324)$
(d) $(5130 + 1410)$ (e) $(3280 + 4395)$ (f) $(10083 + 29380)$

7. Estimate each sum of the nearest thousand :
(a) $(32836 + 16466)$ (b) $(46703 + 11375)$ (c) $(10083 + 29380)$

8. Estimate each difference to the nearest ten :
(a) $(97 - 38)$ (b) $(53 - 18)$ (c) $(409 - 148)$

9. Estimate each difference to the nearest hundred :
(a) $(957 - 578)$ (b) $(678 - 215)$ (c) $(5612 - 3095)$

10. Estimate each difference to the nearest thousand :
(a) $(47005 - 39488)$ (b) $(35863 - 27677)$ (c) $(7258 - 2429)$

ESTIMATING THE PRODUCTS

Illustrative Examples

Example 1. *Estimate the product of* 31 *and* 27 *by rounding off each number to the nearest ten.*

Solution: 31 estimated to the nearest ten = 30

27 estimated to the nearest ten = 30

Hence, the required estimation = $(30 \times 30) = 900$

Example 2. *Estimate the product of* 43 *and* 55 *by rounding of each number to the nearest ten.*

Solution: 43 estimated to the nearest ten = 40

55 estimated to the nearest ten = 60

Hence, the required estimation = $(40 \times 60) = 2400$

Example 3. *Estimate the product of* 367×231 *by rounding off each number to the nearest hundered.*

Solution: 367 estimated to the nearest hundred = 400

231 estimated to the nearest hundred = 200

Hence, the estimated product = $400 \times 200 = 80000$

Example 4. *Estimate the product of* 183 × 153 *by rounding off the first number upwards and the second number downwards.*

Solution: 183 estimated upwards = 200

153 estimated downwards = 100

Hence, the estimated product = 200 × 100 = 20000

Exercise 1E

1. Estimate each of the following products by rounding off each number to the nearest ten :

(a) 57 × 42 (b) 44 × 38 (c) 37 × 52

(d) 61 × 85 (e) 53 × 47 (f) 12 × 25

2. Estimate each of the following products by rounding off each number to the nearest hundred :

(a) 423 × 158 (b) 376 × 123 (c) 264 × 147

3. Estimate each of the following products by rounding off the first number upwards and the second number downwards :

(a) 359 × 76 (b) 267 × 146 (c) 183 × 154

4. Estimate each of the following products by rounding off the first number downwards and the second number upwards :

(a) 578 × 369 (b) 472 × 76 (c) 356 × 278

ESTIMATING THE QUOTIENTS

Illustrative Examples

Example. Find the estimated quotient for each of the following :

(a) 74 ÷ 34 (b) 627 ÷ 23 (c) 985 ÷ 48 (d) 694 ÷ 58

Solution: (a) 74 ÷ 34 is approximately equal to 70 ÷ 30 = 7 ÷ 3, which is approximately equal to 2.

(b) 627 ÷ 23 is approximately equal to 600 ÷ 20 = 30

(c) 985 ÷ 48 is approximately equal to 1000 ÷ 50 = 20

(d) 694 ÷ 58 is approximately equal to 700 ÷ 60, which is approximately equal to 70 ÷ 6 = 12

Exercise 1F

Find the estimated quotient for each of the following :

1. 83 ÷ 17 **2.** 75 ÷ 23 **3.** 87 ÷ 28

4. 725 ÷ 23 **5.** 275 ÷ 25 **6.** 193 ÷ 24

7. 929 ÷ 29 **8.** 633 ÷ 33 **9.** 868 ÷ 38

10. 858 ÷ 39

ROMAN NUMERALS

The Roman system of numeration is one of the most ancient systems of writing numerals which is still in use. We can see Roman numerals on the dial of a clock, on the sizes of clothes, on the numbering of the questions, etc.

There are seven basic symbols used in Roman numerals to write any numeral.

Roman numeral	I	V	X	L	C	D	M
Hindu-Arabic numeral	1	5	10	50	100	500	1000

If a bar is placed over a numeral, it is multiplied by 1000.

Thus, $\overline{V} = 5000$ and $\overline{X} = 10000$, etc.

Using these symbols, we may form all Roman numerals by adopting the rules given below.

Rule 1. Repetition of a symbol in a Roman numeral means addition.

Cautions : (i) Only I, X, C, M can be repeated.
(ii) V, L and D are never repeated.
(iii) No symbol in a Roman numeral can be repeated more than 3 times.

Examples. (a) $III = (1 + 1 + 1) = 3$
(b) $XXX = (10 + 10 + 10) = 30$
(c) $CCC = (100 + 100 + 100) = 300$

Rule 2. A smaller numeral written to the right of a larger numeral is always added to the larger numeral.

Examples. (a) $VII = (5 + 2) = 7$
(b) $XV = (10 + 5) = 15$
(c) $LX = (50 + 10) = 60$

Rule 3. A smaller numeral written to the left of a larger numeral is always subtracted from the larger numeral.

Cautions : (i) V, L and D are never subtracted.
(ii) I can be subtracted from V and X only.
(iii) X can be subtracted from L and C only.
(iv) C can be subtracted from D and M only.

Examples. (a) $IV = (5 - 1) = 4$ (b) $IX = (10 - 1) = 9$
(c) $XL = (50 - 10) = 40$ (d) $XC = (100 - 10) = 90$
(e) $CD = (500 - 100) = 400$ (f) $CM = (1000 - 100) = 900$

Rule 4. When a smaller numeral is placed between two larger numerals, it is always subtracted from the larger numeral immediately following it.

Examples. (a) $XIV = 10 + (5 - 1) = 14$
(b) $XIX = 10 + (10 - 1) = 19$
(c) $CXIV = 100 + 10 + (5 - 1) = 114$

Solved Examples

Example 1. *Write Roman numeral for each of the numbers from 1 to 20.*

Solution: We may write these numbers as given below :

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X
11	12	13	14	15	16	17	18	19	20
XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX

Example 2. *Expres each of the following numbers as a Roman numeral :*

(a) 22 (b) 27 (c) 29 (d) 33
(e) 37 (f) 39 (g) 40 (h) 46
(i) 49 (j) 53 (k) 63 (l) 73
(m) 79 (n) 84 (o) 89 (p) 90
(q) 93 (r) 99 (s) 100

Solution: We may write these numbers as given below :

(a) 22 = XXII (b) 27 = XXVII (c) 29 = XXIX (d) 33 = XXXIII
(e) 37 = XXXVII (f) 39 = XXXIX (g) 40 = XL (h) 46 = XLVI
(i) 49 = XLIX (j) 53 = LIII (k) 63 = LXIII (l) 73 = LXXIII
(m) 79 = LXXIX (n) 84 = LXXXIV (o) 89 = LXXXIX (p) 90 = XC
(q) 93 = XCIII (r) 99 = XCIX (s) 100 = C

Example 3. *Expres each of the following numbers as a Roman numeral :*

(a) 137 (b) 175 (c) 198 (d) 236
(e) 343 (f) 389 (g) 400 (h) 479
(i) 557 (j) 596 (k) 625 (l) 769

Solution: We may write these numbers as given below :

(a) 137 = 100+30+7 = CXXXVII (b) 175 = 100+70+5 = CLXXV
(c) 198 = 100+90+8 = CXCVIII (d) 236 = 200+30+6 = CCXXXVI
(e) 343 = 300+40+3 = CCCXLIII (f) 389 = 300+80+9 = CCCLXXXIX
(g) 400 = CD (h) 479 = 400+ 70+9 = CDLXXIX
(i) 557 = 500 +50 + 7 = DLVII (j) 596 = 500+90+6 = DXCVI
(k) 625 = 500+100+20+5 =DCXXV (l) 769 = 500+200+60+9=DCCLXIX

Example 4. *Expres each of the following as Hindu-Arabic numeral :*

(a) XXIV (b) XLVI (c) LXXXVI (d) XCIX
(e) CLXVI (f) CCXXVI (g) CCCXL (h) CDXLVI

Solution: We may write these numerals as given below :

(a) XXIV = 20 + (5 – 1) = 24
(b) XLVI = XL + VI = (50 – 10) + (5 + 1) = 46
(c) LXXXVI = 50 + 30 + 6 = 86
(d) XCIX = (100 – 10) + (10 – 1) = 99
(e) CLXVI = 100 + 50 + 10 + (5 + 1) = 166

(f) CCXXVI = 200 + 20 + (5 + 1) = 226

(g) CCCXL = 300 + (50 − 10) = 340

(h) CDXLVI = (500 − 100) + (50 − 10) + (5 + 1) = 446

Example 5. *Show that each of the following is meaningless. Give reason in each case.*

(a) XXXX (b) VX (c) IC (d) XVV

Solution: (a) No symbol is repeated more than three times.

∴ XXXX is wrong.

(b) V, L, D are never subtracted.

∴ VX is wrong.

(c) I can be subtracted from V and X only.

∴ IC is wrong.

(d) V, L, D are never repeated.

∴ XVV is wrong.

Exercise 1G

1. Express each of the following as a Roman numeral :

(a) 3	(b) 7	(c) 15	(d) 28
(e) 39	(f) 45	(g) 56	(h) 63
(i) 72	(j) 80	(k) 92	(l) 97
(m) 99	(n) 110	(o) 125	

2. Express each of the following as a Roman numeral :

(a) 154	(b) 185	(c) 230	(d) 343
(e) 485	(f) 595	(g) 613	(h) 757

3. Write each of the following as Hindu-Arabic numeral :

(a) XXVII	(b) XXXIV	(c) XLV	(d) LIV
(e) LXXXIV	(f) XCI	(g) XCVI	(h) CXI
(i) CLIV	(j) CCXXIV	(k) CCCLXV	(l) CDXIV
(m) CDLXIV	(n) DVI	(o) DCCLXVI	

4. Show that each of the following is meaningless. Give reason in each case.

(a) VC (b) IL (c) VVII (d) IXX

Hints

- V is never subtracted.
- I can be subtracted from V and X only.
- V, L, D are never repeated.
- IX cannot occur to the left of X.

Exercise 1H

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. The smallest counting number is :
(a) 0 (b) 1 (c) 10 (d) none of these

2. The difference between the place value and the face value of 5 in the numeral 78653421 is :
(a) 53416 (b) 4995 (c) 49995 (d) none of these

3. The face value of 4 in the numeral 89247605 is :
(a) 4 (b) 40000 (c) 47605 (d) 8924

4. The place value of 6 in the numeral 48632950 is :
(a) 6 (b) 632950 (c) 600000 (d) 486

5. How many 7-digit numbers are there ?
(a) 8999999 (b) 9000000 (c) 1000000 (d) none of these

6. How many 4-digit numbers are there ?
(a) 8999 (b) 9000 (c) 8000 (d) none of these

7. What comes just before 1000000 ?
(a) 99999 (b) 999999 (c) 9999999 (d) none of these

8. How many 8-digit numbers are there ?
(a) 99999999 (b) 89999999 (c) 90000000 (d) none of these

9. Which of the following is not meaningful ?
(a) CI (b) CII (c) IC (d) XC

10. Which of the following is not meaningful ?
(a) VX (b) XV (c) XXV (d) XXXV

THINGS TO REMEMBER

- The place value of any digit depends upon its position in the number.
- The face value of any digit is the digit itself regardless of its position in the number.
- 'Commas' are used in writing large numbers. Commas are not used while writing number names.
- Estimation is the process of finding out an approximate answer to an operation.
- Roman number system uses 7 basic symbols (I, V, X, L, C, D and M) with certain rules to represent any number.

2 Factors and Multiples

In this chapter, we shall review these ideas and extend our study to include some new properties. Here, by numbers we would mean only counting numbers. Recall the following two definitions.

FACTOR : *A factor of a number is a number which divides that number exactly without leaving any remainder.*

MULTIPLE : *A multiple of any natural number is a number formed by multiplying that number by any other number.*

For example : We know that $15 = 1 \times 15$ and $15 = 3 \times 5$.

This shows that each of the numbers 1, 3, 5, 15 exactly divides 15.

Therefore, 1, 3, 5, 15 are all factors of 15.

In other words, we can say that 15 is a multiple of each one of the numbers 1, 3, 5 and 15.

Thus, we conclude that

If a number x divides a number y exactly then x is called a factor of y, and y is called a multiple of x.

Clearly, 1 is a factor of every number.

And, every number is a factor of itself.

It may be noted that 1 is the only number which has exactly one factor, namely, itself.

VARIOUS TYPES OF NUMBERS

(i) EVEN NUMBERS : *The numbers which are multiples of* 2 *or exactly divisible by* 2*, are called even numbers.*

For example : 2, 4, 6, 8, 10, 12, etc., are all even numbers.

(ii) ODD NUMBERS : *The numbers which are not multiples of* 2 *or leave remainder* 1*, on dividing by* 2*, are called odd numbers.*

For example : 1, 3, 5, 7, 9, 11, 13, etc., are all odd numbers.

(iii) PRIME NUMBERS : T*he numbers which have exactly two factors (*1 *and number itself) are called prime numbers.*

For example : 2, 3, 5, 7, 11, 13, 17, 19, 23, etc., are all prime numbers.

(iv) COMPOSITE NUMBERS : *The numbers which have more than two factors are called composite numbers.*

For example : Each of the numbers 4, 6, 8, 9, 10, 12, 14, etc., is a composite number.

IMPORTANT FACTS

(i) 1 is neither prime nor composite.

(ii) 2 is the lowest prime number.

(iii) 2 is the only even prime number. All other even numbers are composite numbers.

FINDING PRIME AND COMPOSITE NUMBERS FROM 1 TO 100

A method for finding the prime and composite numbers from 1 to 100 was found by the Greek mathematician ***Eratosthenes.***

Under this method, we proceed according to the steps given below.

Step 1. Prepare a table of numbers from 1 to 100, taking ten numbers in each row, as shown below.

Step 2. We know that 1 is neither prime nor composite. So, we separate it out by making a square box around it.

Step 3. Encircle ② as a prime number and cross out every multiple of 2.

Step 4. Encircle ③ as a prime number and cross out every multiple of 3. We need not mark the numbers which have already been crossed out.

Step 5. Encircle ⑤ as a prime number and cross out every multiple of 5. We need not mark the numbers which have already been crossed out.

Step 6. Continue this process till the numbers up to 100 are either encircled or crossed-out.

SIEVE OF ERATOSTHENES

1	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Note that :
- All encircled numbers are prime numbers.
- All crossed out numbers are composite numbers.

Thus, all prime numbers from 1 to 100 are :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

TWIN PRIMES : *If the difference between any two prime numbers is 2, then such pair of prime numbers are called twin primes*.

Pairs of twin primes between 1 and 100 are :

(i) 3, 5 (ii) 5, 7 (iii) 11, 13 (iv) 17, 19
(v) 29, 31 (vi) 41, 43 (vii) 59, 61 (viii) 71, 73

PRIME TRIPLET : *A set of three consecutive prime numbers, differing by 2, is called a prime triplet.*
The only prime triplet is (3,5,7).

CO-PRIME : *Two numbers are said to be co-primes if they do not have a common factor other than* 1.
For examples : (i) 2, 3 (ii) 3, 4 (iii) 4, 5 (iv) 4, 9 (v) 8, 15

Note that :
- Two Prime numbers are always co-primes.
- Two co-prime need not be prime numbers.

For examples : (i) 6, 7 are co-primes, while 6 is not a prime number.
(ii) 9, 10 are co-primes, while none of 9 and 10 is prime number.

PERFECT NUMBERS : *If the sum of all the factors of a number is two times the number, then the number is called a perfect number.*
For examples: (i) 6 is a perfect number, since the factors of 6 are 1, 2, 3, 6 and $(1+2+3+6)=(2\times 6)$.
(ii) 28 is a perfect number, since the factors of 28 are 1, 2, 4, 7, 14, 28 and
$(1+2+4+7+14+28)=(2\times 28)$.

Exercise 2A

1. Write down all the factors of :
(a) 20 (b) 36 (c) 60 (d) 75

2. Write the first five multiples of each of the following numbers :
(a) 17 (b) 23 (c) 65 (d) 70

3. Which of the following numbers are even and which are odd?
(a) 42 (b) 47 (c) 60 (d) 68
(e) 79 (f) 174 (g) 231 (h) 352

4. What are prime numbers? Give ten examples.

5. Write all the prime numbers between :
(a) 10 and 30 (b) 35 and 60 (c) 61 and 80 (d) 81 and 100

6. (a) Write the smallest prime number.
(b) List all even prime numbers.
(c) Write the smallest odd prime number.

7. Find which of the following numbers are primes :
(a) 31 (b) 51 (c) 93 (d) 96

8. Make a list of seven consecutive numbers, none of which is prime.
Hint : *See the sieve of Eratosthenes.*

9. What are composite numbers? Can a composite number be odd? If yes, write the smallest odd composite number.

10. What are twin primes? Write all the pairs of twin primes between 50 and 100.

11. What are co-primes? Give examples of five pairs of co-primes. Are co-primes always primes? If no, illustrate your answer by an example.

12. Express each of the following numbers as the sum of two odd primes :
(a) 36 (b) 42 (c) 84 (d) 98

13. Express each of the following odd numbers as the sum of three odd prime numbers :
(a) 31 (b) 35 (c) 49 (d) 63

14. Express each of the following numbers as the sum of twin primes :
(a) 36 (b) 84 (c) 120 (d) 144

15. Which of the following statements are true?

(a) 1 is the smallest prime number.

(b) If a number is prime, it must be odd.

(c) The sum of two prime numbers is always a prime number.

(d) If two numbers are co-primes, at least one of them must be a prime number.

DIVISIBILITY TESTS FOR 2, 3, 4, 5, 6, 7, 8, 9, 10 AND 11

(i) TEST OF DIVISIBILITY BY 2 : *A number is divisible by* 2 *if its ones digit is* 0, 2, 4, 6 *or* 8.

Example 1. Each of the numbers 20, 42, 74, 156, 5008 is divisible by 2.

Example 2. None of the numbers 41, 63, 325, 557, 849 is divisible by 2.

(ii) TEST OF DIVISIBILITY BY 3 : *A number is divisible by* 3 *if the sum of its digits is divisible by* 3.

Example 1. Consider the number 74163.

Sum of its digits $= (7 + 4 + 1 + 6 + 3) = 21$, which is divisible by 3.

Therefore, 74163 is divisible by 3.

Example 2. Consider the number 984602.

Sum of its digits $= (9 + 8 + 4 + 6 + 0 + 2) = 29$, which is divisible by 3.

Therefore, 984602 is not divisible by 3.

(iii) TEST OF DIVISIBILITY BY 4 : *A number is divisible by* 4 *if the number formed by its digits in the tens and ones places is divisible by* 4.

Example 1. Consider the number 57532.

The number formed by the tens and ones digits is 32, which is divisible by 4.

Therefore, 57532 is divisible by 4.

Example 2. Consider the number 37266.

The number formed by the tens and ones digits is 66, which is divisible by 4.

Therefore, 37266 is divisible by 4.

(iv) TEST OF DIVISIBILITY BY 5 : *A number is divisible by* 5 *if its ones digit is* 0 *or* 5.

Example 1. Each of the numbers 65, 195, 230, 310 is divisible by 5.

Example 2. None of the numbers 71, 83, 94, 106, 327, 148, 279 is divisible by 5.

(v) TEST OF DIVISIBILITY BY 6 : *A number is divisible by* 6 *if it is divisible by each one of* 2 *and* 3.

Note that 2 and 3 are the prime factors of 6.

Example 1. Each of the numbers 18, 42, 60, 114, 1356 is divisible by 6.

Example 2. None of the numbers 21, 25, 34, 52 is divisible by 6.

(vi) TEST OF DIVISIBILITY BY 7 : *A number is divisible by* 7 *if the difference between twice the ones digit and the number formed by the other digits is either* 0 *or a multiple of* 7.

Example 1. Consider the number 6804.

Clearly, $(680 - 2 \times 4) = 672$, which is divisible by 7.

Therefore, 6804 is divisible by 7.

Example 2. Consider the number 137.

Clearly, $(2 \times 7) - 13 = 1$, which is not divisible by 7.

Therefore, 137 is not divisible by 7.

Example 3. Consider the number 1367.

Clearly, $136 - (2 \times 7) = 136 - 14 = 122$, which is not divisible by 7.

Therefore, 1367 is not divisible by 7.

(vii) TEST OF DIVISIBILITY BY 8 : *A number is divisible by* 8 *if the number formed by its digits in hundreds, tens and ones places is divisible by* 8.

Example 1. Consider the number 516664.

The number formed by hundreds, tens and ones digits is 664, which is clearly divisible by 8.

Therefore, 516664 is divisible by 8.

Example 2. Consider the number 65148.

The number formed by hundreds, tens and ones digits is 148, which is not divisible by 8.

Therefore, 65148 is not divisible by 8.

(viii) TEST OF DIVISIBILITY BY 9 : *A number is divisible by* 9 *if the sum of its digits is divisible by* 9.

Example 1. Consider the number 756981.

Sum of its digits $= (7 + 5 + 6 + 9 + 8 + 1) = 36$, which is divisible by 9.

Therefore, 756981 is divisible by 9.

Example 2. Consider the number 92437.

Sum of its digits $= (9 + 2 + 4 + 3 + 7) = 25$, which is not divisible by 9.

Therefore, 92437 is not divisible by 9.

(ix) TEST OF DIVISIBILITY BY 10 : *A number is divisible by* 10 *if its ones digit is* 0.

Example 1. Each of the numbers 30, 160, 690, 720 is divisible by 10.

Example 2. None of the numbers 21, 32, 63, 84, etc., is divisible by 10.

(x) TEST OF DIVISIBILITY BY 11 : *A number is divisible by* 11 *if the difference of the sum of its digits in odd places and the sum of its digits in even places* (*starting from the ones place*) *is either* 0 *or a multiple of* 11.

Example 1. Consider the number 90728.

Sum of its digits in odd places $= (8 + 7 + 9) = 24$

Sum of its digits in even places $= (2 + 0) = 2$

Difference of the two sums $= (24 - 2) = 22$, which is clearly divisible by 11.

Therefore, 90728 is divisible by 11.

Example 2. Consider the number 863423.

Sum of its digits in odd places $= (3 + 4 + 6) = 13$

Sum of its digits in even places $= (2 + 3 + 8) = 13$

Difference of the two sums $= (13 - 13) = 0$

Therefore, 863423 is divisible by 11.

Example 3. Consider the number 76844.

Sum of its digits in odd places $= (4 + 8 + 7) = 19$

Sum of its digits in even places $= (4 + 6) = 10$

Difference of these sums $= (19 - 10) = 9$, which is not divisible by 11.

Therefore, 76844 is not divisible by 11.

GENERAL PROPERTIES OF DIVISIBILITY

PROPERTY 1. *If a number is divisible by another number, it must be divisible by each of the factors of that number.*

Example We know that 36 is divisible by 12.
All factors of 12 are 1, 2, 3, 4, 6, 12.
Clearly, 36 is divisible by each one of 1, 2, 3, 4, 6, 12.

Remarks

As a consequence of the above result, we can say that
(i) every number divisible by 9 is also divisible by 3.
(ii) every number divisible by 8 is also divisible by 4.

PROPERTY 2. *If a number is divisible by each of two co-prime number, it must be divisible by their product.*

Example 1. We know that 972 is divisible by each of the numbers 2 and 3. Also, 2 and 3 are co-primes.
So, according to Property 2, the number 972 must be divisible by 6, which is true.

Example 2. We know that 4320 is divisible by each one of the numbers 5 and 8. Also, 5 and 8 are co-primes.
So, 4320 must be divisible by 40.
By actual division, we find that it is true.

Example 3. Consider the number 372.
It may be verified that the above number is divisibly by both 4 and 6.
But, by actual division, we find that 372 is not divisible by 24.
By careful, 4 and 6 are not co-primes.

Remark

Since two prime numbers are always co-primes, it follows that if a number is divisible by each one of any two prime numbers then the number is divisible by their product.

PROPERTY 3. *If a number is a factor of each of two given numbers, then it must be a factor of their sum.*

Example 1. We know that 5 is a factor of 15 as well as that of 20.
So, 5 must be a factor of $(15 + 20)$, that is 35.
And, this is clearly true.

Example 2. We know that 7 is a factor of each of the numbers 49 and 63.
So, 7 must be a factor of $(49 + 63) = 112$.
Clearly, 7 divides 112 exactly.

PROPERTY 4. *If a number is a factor of each of two given numbers then it must be a factor of their difference.*

Example 1. We know that 3 is a factor of each one of the numbers 36 and 24.
So, 3 must be a factor of $(36 - 24) = 12$.
Clearly, 3 divides 12 exactly.

Example 2. We know that 13 is a factor of each one of the numbers 65 and 117.
So, 13 must be a factor of $(117 - 65) = 52$.
Clearly, 13 divides 52 exactly.

TO FIND PRIME NUMBERS BETWEEN 100 AND 200

We know that $15 \times 15 > 200$.

So, we adopt the following rule :

Rule *Examine whether the given number is divisible by any prime number less than* 15. *If yes then it is not prime, otherwise it is prime.*

EXAMPLE *Which of the following are prime numbers*?

(a) 117 (b) 139 (c) 193

Solution: (a) Test the divisibility of 117 by each one of the prime numbers 2, 3, 5, 7, 11, 13, taking one by one. We find that 117 is divisibly by 13.

So, 117 is not a prime number.

(b) Test the divisibility of 139 by each one of the prime numbers 2, 3, 5, 7, 11, 13. We find that 139 is divisible by none of them.

So, 139 is a prime number.

(c) Test the divisibility of 193 by each one of the prime numbers 2, 3, 5, 7, 11, 13.

We find that 193 is divisible by none of them.

So, 193 is a prime number.

TO FIND PRIME NUMBERS BETWEEN 100 AND 400

We know that $20 \times 20 = 400$.

Rule: *Examine whether the given number is divisible by any prime number less than* 20. *If yes then it is not prime, otherwise it is prime.*

EXAMPLE: *Which of the following is a prime numbers*?

(a) 263 (b) 323 (c) 361

Solution: (a) Test the divisibility of 263 by each one of the prime numbers 2, 3, 5, 7, 11, 13, 17, 19. We find that 263 is not divisible by any of these numbers.

So, 263 is a prime number.

(b) Test the divisibility of 323 by each one of the prime numbers 2, 3, 5, 7, 11, 13, 17 19. We find that 323 is divisible by 17.

So, 323 is not a prime number.

(c) Test the divisibility of 361 by each one of the prime numbers 2, 3, 5, 7, 11, 13, 17 19. We find that 361 is divisible by 19.

So, 361 is not a prime number.

Exercise 2B

1. Test the divisibility of the following numbers by 2 :

(a) 168 (b) 8370 (c) 63921 (d) 367314

(e) 357986 (f) 789403

2. Test the divisibility of the following numbers by 3 :

(a) 533 (b) 20701 (c) 10038 (d) 872645

(e) 79124 (f) 524781

3. Test the divisibility of the following numbers by 4 :

(a) 738 (b) 3314 (c) 72712 (d) 25064

(e) 835226 (f) 720832

4. Test the divisibility of the following numbers by 5 :

(a) 2850 (b) 27485 (c) 28506 (d) 834505

(e) 245792 (f) 549860

5. Test the divisibility of the following numbers by 6 :

(a) 3030 (b) 17852 (c) 61233 (d) 639210

(e) 951480 (f) 6839452

6. Test the divisibility of the following numbers by 7 :

(a) 117 (b) 826 (c) 6021 (d) 2345

(e) 25368 (f) 14126

7. Test the divisibility of the following numbers by 8 :

(a) 2138 (b) 9364 (c) 901674 (d) 36792

(e) 1790184 (f) 136976

8. Test the divisibility of the following numbers by 9 :

(a) 3333 (b) 2358 (c) 98712 (d) 326999

(e) 647514 (f) 257106

9. Test the divisibility of the following numbers by 10 :

(a) 6870 (b) 52325 (c) 44550 (d) 43238

10. Test the divisibility of the following numbers by 11 :

(a) 7678 (b) 66311 (c) 83721 (d) 901351

(e) 137269 (f) 8790322

11. In each of the following numbers, replace * by the smallest number to make it divisible by 3 :

(a) 27*4 (b) 8*711 (c) 53*46 (d) 6*1054

(e) 234*17 (f) 62*35

12. In each of the following numbers, replace * by the smallest number to make it divisible by 9 :

(a) 65*5 (b) 6702* (c) 2*135 (d) 6678*1

(e) 91*67 (f) 835*86

13. Which of the following are prime numbers?

(a) 103 (b) 137 (c) 161 (d) 179

14. Give an example of a number :

(a) which is divisible by 2 but not by 4.

(b) which is divisible by 4 but not by 8.

(c) which is divisible by both 2 and 8 but not by 16.

(d) which is divisible by both 3 and 6 but not by 18.

15. Write (T) for true and (F) for false against each of the following statements :

(a) If a number divides the sum of two numbers exactly, it must exactly divide the numbers separately.

(b) If a number is divisible by 3 and 7, it must be divisible by 21.

(c) The sum of two consecutive odd numbers is always divisible by 4.

(d) If a number divides two numbers exactly, it must divide their sum exactly.

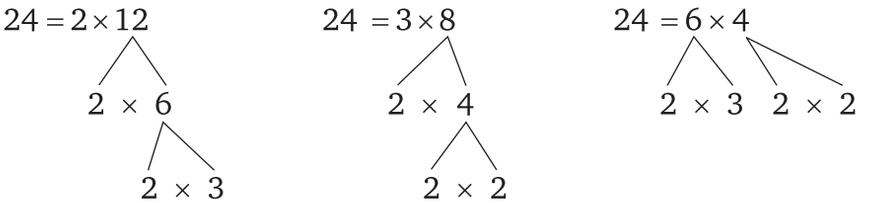
PRIME FACTORIZATION

PRIME FACTOR : *A factor of a given number is called a prime factor if this factor is a prime number.*

Example: 2 *and* 3 *are prime factors of* 12.

PRIME FACTORIZATION : The process of expressing a number as a product of its factors is called factorization.

Example : *Let us factorize* 24 *in three different ways as given below* :



Thus, $36 = 2\times2\times2\times3$; $36 = 3\times2\times2\times2$; $36 = 2\times3\times2\times2$

We notice here that in each of the prime factorizations, the factors may be arranged differently but, in fact, they are the same.

Thus, we generalise this result as under.

Every composite number can be factorized into primes in only one way, except for the order of primes.

This property is known as unique factorization property.

CONCEPT OF POWER

We write, $2\times2 = 2^2$ (read as 2 raised to the power 2),

$2\times2\times2 = 2^3$ (read as 2 raised to the power 3),

$2\times2\times2\times2 = 2^4$ (read as 2 raised to the power 4),

and so on.

Similarly, $3\times3 = 3^2, 3\times3\times3 = 3^3, 3\times3\times3\times3 = 3^4$ and so on.

In general, $a\times a\times \ldots$ taken n times $= a^n$.

Solved Examples

Example 1. *Give the prime factorization of* 504.

Solution: We use the division method, as shown below.

2	504
2	252
2	126
3	63
3	21
7	7
	1

$\therefore\ 504 = 2\times2\times2\times3\times3\times7 = 2^3\times3^2\times7$

Example 2. *Give the prime factorization of* 70070.

Solution: We have :

2	70070
5	35035
7	7007
7	1001
11	143
13	13
	1

Therefore, $70070 = 2 \times 5 \times 7 \times 7 \times 11 \times 13 = 2 \times 5 \times 7^2 \times 11 \times 13$

Exercise 2C

Give the prime factorization of each of the following numbers :

1. 28	**2.** 40	**3.** 85	**4.** 96
5. 120	**6.** 140	**7.** 375	**8.** 480
9. 625	**10.** 980	**11.** 1024	**12.** 3125
13. 4335	**14.** 4641	**15.** 2907	**16.** 8712
17. 1323	**18.** 9317	**19.** 8712	**20.** 17424

HCF AND LCM

HIGHEST COMMON FACTOR (HCF) : *The greatest number which is a common factor of two or more given numbers, is called their* ***highest common factor*** *or* ***greatest common divisor*** *or* ***greatest common measure,*** *written as HCF or GCD or GCM.*

EXAMPLE: *Let us find the HCF of* 24 *and* 32.

Solution: All the factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

All the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36

Common factors of 24 and 36 are: 1, 2, 3, 4, 6, 12

Thus, the highest common factor of 24 and 36 is 12.

Hence, HCF of 24 and 36 = 12

TO FIND HCF (BY PRIME FACTORIZATION METHOD) : *We first find the prime factorization of each of the given numbers. Then, the product of all common prime factors, using the least power of each common prime factor, is the HCF of the given numbers.*

Solved Examples

Example 1. *Find the HCF of* 112 *and* 168 *by the prime factorization method.*

Solution: We have,

2	112
2	56
2	28
2	14
7	7
	1

2	168
2	84
2	42
3	21
7	7
	1

$112 = 2 \times 2 \times 2 \times 2 \times 7 = 2^4 \times 7$

And, $168 = 2\times2\times2\times3\times7 = 2^3\times3\times7$

HCF of 112 and $168 = 2^3\times7 = 56$

Example 2. *Find the HCF of* 144, 180 *and* 192 *by the prime factorization method.*

Solution: We have,

2 | 144
2 | 72
2 | 36
2 | 18
3 | 9
3 | 3
| 1

2 | 180
2 | 90
3 | 45
3 | 15
5 | 5
| 1

2 | 192
2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3 | 3
| 1

So, $144 = 2\times2\times2\times2\times3\times3 = 2^4\times3^2$;

$180 = 2\times2\times3\times3\times5 = 2^2\times3^2\times5$;

$192 = 2\times2\times2\times2\times2\times2\times3 = 2^6\times3$.

Therefore, the HCF of the given numbers $= 2^2\times3 = 12$

TO FIND HCF (BY DIVISION METHOD) : *Suppose two numbers are given. Divide the greater number by the smaller one. Next, divide the divisor by the remainder. Go on repeating the process of dividing the preceding divisor by the remainder last obtained till the remainder zero is obtained. Then the last divisor is the required HCF of the given numbers.*

Example 3. *Find the HCF of* 161 *and* 345 *by the division method.*

Solution: We have,

$$\begin{array}{l} 161\overline{)345}\,(2 \\ \quad -322 \\ \hline \quad 23\overline{)161}\,(7 \\ \quad\quad -161 \\ \hline \quad\quad\quad \times \end{array}$$

Hence, the HCF of 161 and 345 is 23.

TO FIND THE HCF OF MORE THAN TWO NUMBERS : *If more than two numbers are given, choose any two of them and find their HCF. The HCF of this HCF and the third number gives the HCF of these three numbers. The HCF of this HCF and the fourth number gives the HCF of these four numbers and so on.*

Example 4. *Find the HCF of* 136, 170 *and* 255.

Solution: First we find the HCF of 136 and 170.

$$\begin{array}{l} 136\overline{)170}\,(1 \\ \quad -136 \\ \hline \quad 34\overline{)136}\,(4 \\ \quad\quad -136 \\ \hline \quad\quad\quad \times \end{array}$$

Thus, the HCF of 136 and 170 is 34.

Now, we find the HCF of 34 and 255.

$$\begin{array}{l} 34\overline{)255}\,(7 \\ \quad -238 \\ \hline \quad 17\overline{)34}\,(2 \\ \quad\quad -34 \\ \hline \quad\quad\quad \times \end{array}$$

So, the HCF of 34 and 255 is 17.
Hence, the HCF of 136, 170 and 255 is 17.

Example 5. *Find the greatest number which divides* 285 *and* 1249*, leaving remainders* 9 *and* 7 *respectively.*

Solution: Clearly, we must find the greatest number which divides (285 – 9) and (1249 – 7) exactly.
So, the required number = HCF of 276 and 1242.

$$\begin{array}{r} 276\overline{)1242}(4 \\ -1104 \\ \hline 138\overline{)276}(2 \\ -276 \\ \hline \times \end{array}$$

Hence, the required number = 138

Example 6. *Reduce* $\frac{289}{391}$ *to the lowest terms.*

Solution: For reducing the given fraction to the lowest terms, we divide its numerator and the denominator by their HCF.
Now, we find the HCF of 289 and 391 as under.

$$\begin{array}{r} 289\overline{)391}(1 \\ -289 \\ \hline 102\overline{)289}(2 \\ -204 \\ \hline 85\overline{)102}(1 \\ -85 \\ \hline 17\overline{)85}(5 \\ -85 \\ \hline \times \end{array}$$

Hence, the HCF of 289 and 391 is 17.
Now, dividing the numerator and the denominator of the given fraction by 17, we get

$$\frac{289}{391} = \frac{289 \div 17}{391 \div 17} = \frac{17}{23}$$

Example 7. *The length, breadth and height of a room are* 1050 *cm,* 750 *cm and* 425 *cm respectively. Find the maximum length of the tape which can measure the three dimensions of the room exactly.*

Solution: The maximum length of the tape which can measure the given dimensions
= HCF of 1050 cm, 750 cm and 425 cm
First we find the HCF of 1050 and 750.

$$\begin{array}{r} 750\overline{)1050}(1 \\ -750 \\ \hline 300\overline{)750}(2 \\ -600 \\ \hline 150\overline{)300}(2 \\ -300 \\ \hline \times \end{array}$$

Hence, HCF of 1050 and 750 = 150

Now, we find the HCF of 150 and 425.

$$\begin{array}{r} 150\overline{)425}(2 \\ -\,300 \\ \hline 125\overline{)150}(1 \\ -\,125 \\ \hline 25\overline{)125}(5 \\ -\,125 \\ \hline \times \end{array}$$

Hence, HCF of 150 and 425 is 25.
Thus, the HCF of 1050, 750 and 425 is 25.
Hence, the required maximum length = 25 cm

Exercise 2D

1. Find the HCF of the numbers in each of the following, using the prime factorization method :
(a) 25, 15 (b) 16, 56 (c) 24, 42 (d) 16, 18
(e) 33, 99 (f) 25, 45 (g) 18, 45, 72 (h) 21, 42, 70

2. Find the HCF of the numbers in each of the following, using the division method:
(a) 60, 80 (b) 140, 168 (c) 72, 84 (d) 60, 96, 150
(e) 91, 49, 112 (f) 75, 100, 140 (g) 72, 144, 252 (h) 144, 180, 192

3. Show that the following pairs are co-primes:
(a) 59, 97 (b) 385, 621 (c) 512, 945 (d) 161, 192
(e) 343, 432 (f) 847, 1014
[*Hint*. *Two numbers are co-primes if their HCF is* 1].

4. Find the greatest number that will divide 445, 572 and 699, leaving remainders 4, 5, 6 respectively.

5. Find the greatest number which divides 615 and 963, leaving the remainder 6 in each case.

6. Find the greatest number which divides 2011 and 2623, leaving remainders 9 and 5 respectively.

7. Reduce each of the following fractions to the lowest terms :
(a) $\frac{161}{207}$ (b) $\frac{296}{481}$ (c) $\frac{517}{799}$

8. Three different containers contain 403 l, 434 l and 465 l of milk respectively. Find the capacity of a container which can measure the milk of all the containers in an exact number of times.

9. A rectangular courtyard is 18 m 72 cm long and 13 m 20 cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.

LOWEST COMMON MULTIPLE (LCM) : The lowest common multiple of two or more numbers is the lowest or smallest of their common multiple.

Example: *Let us find the LCM of* 8 *and* 12.

Solution: Multiples of 8 are : 8, 16, 24, 32, 40.....
Multiples of 12 are : 12, 24, 36, 48 .....
Common multiples of 8 and 12 are : 24, 48 .....
Lowest common multiple of 8 and 12 is 24.
Hence, LCM of 8 and 12 = 24

TO FIND LCM (BY PRIME FACTORIZATION METHOD) : *In order to find the LCM of two or more given numbers, we write the prime factorization of each of the given numbers. Then, the required LCM of these numbers is the product of all different prime factors of the numbers, using the greatest power of each common prime factor.*

Solved Examples

Example 1. *Find the LCM of* 12, 48 *and* 60 *by the prime factorization method.*

Solution: We have,

2 | 12
2 | 6
3 | 3
| 1

2 | 48
2 | 24
2 | 12
2 | 6
3 | 3
| 1

2 | 60
2 | 30
3 | 15
5 | 5
| 1

$\therefore \quad 12 = 2^2 \times 3$

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

Hence, the LCM of 12, 48 and 60 is $2^4 \times 3 \times 5 = 240$

TO FIND LCM (BY DIVISION METHOD): *In this method, we arrange the given numbers in a line, in any order. We divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. This process is repeated till no two of the given numbers are divisible by a common number. The product of the divisors and the undivided numbers is the required LCM of the given numbers.*

Example 2. *Find the LCM of* 84, 126 *and* 288 *by the division method.*

Solution: We have,

2 | 84, 126, 288
2 | 42, 63, 144
3 | 21, 63, 72
3 | 7, 21, 24
7 | 7, 7, 8
| 1, 1, 8

Hence, the LCM of the given numbers $= 2 \times 2 \times 3 \times 3 \times 7 \times 8 = 2016$

Example 3. *Find the smallest number which when diminished by* 3 *is divisible by* 21, 28, 36 *and* 45.

Solution: We know that the smallest number divisible by 21, 28, 36 and 45 is their LCM.

We calculate this LCM as under,

7 | 21, 28, 36, 45
3 | 3, 4, 36, 45
3 | 1, 4, 12, 15
4 | 1, 4, 4, 5
| 1, 1, 1, 5

Hence, the LCM of 21, 28, 36 and 45 is $7 \times 3 \times 3 \times 4 \times 5 = 1260$.

Hence, the required number $= (1260 + 3) = 1263$

Example 4. *In a shop, there are three clocks which chime at intervals of* 15, 20 *and* 30 *minutes respectively. They all chime together at* 10 *a.m. At what time will they all chime together again?*

Solution: Required time = LCM of 15, 20, 30 minutes

$\therefore$ LCM of 15, 20, 30 $= (5 \times 3 \times 2 \times 2) = 60$

So, all the clocks will chime together again after 60 minutes, *i.e.*, after 1 hour, *i.e.*, at 11 a.m.

5	15, 20, 30
3	3, 4, 6
2	1, 4, 2
	1, 2, 1

PROPERTIES OF HCF AND LCM OF GIVEN NUMBERS

(i) *The HCF of a group of numbers is not greater than any of the given numbers.*

(ii) *The HCF of two co-primes is* 1.

(iii) *The LCM of a group of number is not less than any of the given numbers.*

(iv) *The LCM of two co-primes is equal to their product.*

(v) *The HCF of a group of numbers is always a factor of their LCM.*

Example: Consider the numbers 12, 16, 36, 40.

Clearly, the HCF of the given numbers = 4

And, their LCM $= 2 \times 2 \times 3 \times 2 \times 2 \times 3 \times 5 = 720$

Clearly, 4 is a factor of 720.

2	12, 16, 36, 40
2	6, 8, 18, 20
3	3, 4, 9, 10
2	1, 4, 3, 10
	1 2, 3, 5

(vi) *If a and b are two given numbers such that a is a factor of b then their HCF = a and their LCM = b.*

Example: We know that 8 is a factor of 32.

Then, clearly HCF of 8 and 32 is 8.

And, LCM of 8 and 32 is 32.

(vii) *If two numbers are given then*

product of the two numbers = product of their HCF and LCM

Example: Consider the numbers 48 and 60.

We have, $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

And, $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

So, the HCF of 48 and 60 is $2^2 \times 3 = 12$.

And, the LCM of 48 and 60 is $2^4 \times 3 \times 5 = 240$.

Now, the product of the given numbers $= 48 \times 60 = 2880$

Product of their HCF and LCM $= (12 \times 240) = 2880$

$\therefore$ product of two numbers = (their HCF) × (their LCM)

Remarks

Thus, for any two given numbers, we have

(i) $\text{LCM} = \dfrac{(\text{one number}) \times (\text{the other number})}{\text{their HCF}}$

(ii) $\text{HCF} = \dfrac{(\text{one number}) \times (\text{the other number})}{\text{their LCM}}$

Example 5. *Find the HCF and the LCM of* 1152 *and* 1664.

Solution: We first find the HCF of the given numbers.

$$\begin{array}{r|l} 1152 & 1664 \\ -1152 & 1 \\ \hline 512 & 1152 \\ -1024 & 2 \\ \hline 128 & 512 \\ 512 & 4 \\ \hline \times & \end{array}$$

$\therefore$ HCF = 128

And, $\text{LCM} = \dfrac{\text{product of the numbers}}{\text{their HCF}} = \dfrac{1152 \times 1664}{128} = 14976$

$\therefore$ HCF = 128 and LCM = 14976.

Example 6. *The HCF of two numbers is* 13 *and their product is* 1989. *Find their LCM.*

Solution: We know that

$$\text{LCM} = \frac{\text{product of the given two numbers}}{\text{their HCF}} = \frac{1989}{13} = 153$$

Example 7. *The HCF of two numbers is* 29 *and their LCM is* 1160. *If one of the numbers is* 145, *find the other.*

Solution: We know that

one number $\times$ the other number = HCF $\times$ LCM

Hence, the required number $= \left(\dfrac{29 \times 1160}{145}\right) = 232$

Example 8. *Can two numbers have* 16 *as their HCF and* 204 *as their LCM? Given reason.*

Solution: We know that the HCF of two or more numbers must divide their LCM exactly.

But, 16 does not divide 204 exactly.

So, there can be no two numbers with 16 as their HCF and 204 as their LCM.

Exercise 2E

1. Find the LCM of the numbers given below:

(a) 15, 20 (b) 18, 30 (c) 7, 14, 28 (d) 28, 70, 84

(e) 18, 36, 60, 72 (f) 20, 25, 30, 50 (g) 48, 56, 105, 225 (h) 8, 12, 20, 30, 80

(i) 9, 12, 18, 24, 27 (j) 35, 105, 140, 280

2. Find the HCF and LCM of :

(a) 117, 221 (b) 234, 572 (c) 693, 1078 (d) 145, 232

3. For each pair of numbers, verify that their product = (HCF $\times$ LCM).

(a) 87, 145 (b) 186, 403 (c) 490, 1155

4. The HCF and LCM of two numbers are 131 and 8253 respectively. If one of the numbers is 917, find the other.

5. The HCF of two numbers is 145 and their LCM is 2175. If one of the the numbers is 725, find the other.

6. The product of two numbers is 2160 and their HCF is 12. Find their LCM.

7. The product of two numbers is 2560 and their LCM is 320. Find their HCF.
8. Find the least number which when divided by 25, 40 and 60 leaves 9 as the remainder in each case.
9. Find the greatest number of five digits exactly divisibly by 9, 12, 15, 18 and 24.
10. Find the least number of five digits that is exactly divisible by 16, 18, 24 and 30.
11. Three measuring rods are 45 cm, 50 cm and 75 cm in length. What is the least length (in metres) of a rope that can be measured by the full length of each of these three rods?
12. Three bells toll at intervals of 9, 12, 15 minutes. If they start tolling together, after what time will they next toll together?
13. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds. If they start changing simultaneously at 8 a.m. after how much time will they change again simultaneously?

Exercise 2F

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. Which of the following numbers is divisible by 3?
   (a) 24357806 (b) 35769812 (c) 83479560 (d) 3336433
2. Which of the following numbers is divisible by 4?
   (a) 78653234 (b) 98765042 (c) 24689602 (d) 87941032
3. Which of the following numbers is divisible by 6?
   (a) 8790432 (b) 98671402 (c) 85492014 (d) none of these
4. Which of the following numbers is divisible by 8?
   (a) 96354142 (b) 37450176 (c) 57064214 (d) none of these
5. Which of the following numbers is divisible by 9?
   (a) 8576901 (b) 96345210 (c) 67594310 (d) none of these
6. Which of the following numbers is divisible by 11 ?
   (a) 3333333 (b) 1111111 (c) 22222222 (d) none of these
7. Which of the following is a prime number?
   (a) 323 (b) 361 (c) 263 (d) none of these
8. Which of the following is a prime number?
   (a) 81 (b) 87 (c) 91 (d) 97
9. Which of the following is a prime number?
   (a) 117 (b) 171 (c) 179 (d) none of these
10. Which of the following are co-primes?
    (a) 8, 12 (b) 9, 10 (c) 6, 8 (d) 15, 18
11. Which of the following is a composite number?
    (a) 23 (b) 29 (c) 32 (d) none of these
12. The HCF of 144 and 198 is :
    (a) 12 (b) 16 (c) 18 (d) 8
13. The HCF of 48 and 96 is :
    (a) 9 (b) 12 (c) 16 (d) 18

14. $\frac{289}{391}$ when reduced to the lowest terms is :

(a) $\frac{11}{23}$ (b) $\frac{13}{31}$ (c) $\frac{17}{31}$ (d) $\frac{17}{23}$

15. The LCM of 24, 36, 40 is :

(a) 4 (b) 90 (c) 360 (d) 720

16. The LCM of 12, 15, 20, 27 is :

(a) 270 (b) 360 (c) 480 (d) 540

17. The greatest number which divides 134 and 167 leaving 2 as remainder in each case is :

(a) 14 (b) 17 (c) 19 (d) 33

18. The least number divisible by each of the numbers 15, 20, 24, 32 and 36 is :

(a) 1660 (b) 2880 (c) 1440 (d) none of these

19. The product of two numbers is 2160 and their HCF is 12. The LCM of these numbers is :

(a) 12 (b) 25920 (c) 180 (d) none of these

20. The HCF of two numbers is 145 and their LCM is 2175. If one of the numbers is 725, the other number is :

(a) 290 (b) 435 (c) 5 (d) none of these

THINGS TO REMEMBER

- Suppose a number x divides a number y exactly. Then, we say that x is a factor of y. Also, in this case, we say that y is a multiple of x.
- 1 is the only number having exactly one factor.
- A number having exactly two factors is called a prime number.
- The only even prime number is 2.
- The HCF of two co-primes is 1.
- If x is a factor of y then the HCF of x and y is x, and the LCM of x and y is y.
- The HCF of two or more than two numbers is a factor of their LCM.
- The product of the HCF and LCM of two numbers is equal to the product of the numbers.

ANSWER SHEET

Exercise 1(A)

1. (a) 8014 (b) 43062 (c) 203605 (d) 5020007
(e) 70400037 (f) 60504303 (g) 155020068
(h) 121220012

2. (a) Seventy four thousand six
(b) Eight lakh eight thousand eighty six
(c) Forty-three lakh thirty thousand twenty-eight
(d) Two crore six lakh eight thousand thirteen
(e) Six crore twenty lakh four thousand seven hundred five
(f) Seven crore twenty-nine lakh six thousand nine
(g) Twenty-eight crore eight lakh eight thousand eight hundred
(h) Seven crore twenty-six lakh forty thousand nine hundred eight
(i) Five crore fifty lakh fifty thousand fifty.

3. (a) $2\times10000+6\times1000+6\times100+5\times10+7\times1$
(b) $4\times100000+9\times1000+8\times100+1\times10+6\times1$
(c) $1\times1000000+3\times100000+4\times1000+5\times100+8\times1$
(d) $4\times10000000+2\times1000000+5\times100000+2\times10000+7\times1000+3\times100+8\times10+2\times1$
(e) $8\times10000000+8\times100000+8\times1000+8\times1$
(f) $8\times10000000+2\times1000000+3\times10000+6\times100+2\times10$

4. (a) 53475 (b) 462352 (c) 30205806 (d) 8302903

5. 6999993 **6.** 8999100 **7.** 9000000 **8.** 900000

9. 10000 thousands **10.** 100 thousands **11.** 9999999

12. 9899999 **13.** 9548000 **14.** 99 **15.** 964320

16. 234, 243, 324, 342, 423, 432 **17.** 10357

18.

	HM	TM	M	H Th	T Th	Th	H	T	O
(i)		3	0	1	0	5	0	6	3
(ii)		5	2	2	0	5	0	0	6
(iii)			5	0	0	5	0	0	5

19. International Place-value chart :

	HM	TM	M	H Th	T Th	Th	H	T	O
(a)				7	3	5	8	2	1
(b)			6	0	5	7	8	9	4
(c)		5	6	9	4	3	8	2	1
(d)		3	7	5	0	2	0	9	3
(e)		8	9	3	5	0	0	6	4
(f)		9	0	7	0	3	0	0	6

Number Names:
(a) Seven hundred thirty-five thousand eight hundred twenty-one
(b) Six million fifty-seven thousand eight hundred ninety-four
(c) Fifty-six million nine hundred forty-three thousand eight hundred twenty-one
(d) Thirty-seven million five hundred two thousand ninety-three
(e) Eighty-nine million three hundred fifty thousand sixty-four
(f) Ninety million seven hundred three thousand and six

Exercise 1(B)

1. (a) > (b) < (c) < (d) < (e) > (f) <

2. (a) 990357 < 9873426 < 9874012 < 24615019 < 24620010
(b) 5694437 < 5695440 < 56943201 < 56943300 < 56944000
(c) 700087 < 8014257 < 8014306 < 8015032 < 10012458
(d) 893245 < 893425 < 980134 < 1020216 < 1020304 < 1021403

3. (a) 102345680 > 63521047 > 63514759 > 7355014 > 7354206
(b) 23794206 > 23756819 > 5032790 > 5032786 > 987876
(c) 16060666 > 16007777 > 1808090 > 1808088 > 190909 > 181888
(d) 1712040 > 1704382 > 1702497 > 201200 > 200175 > 199988

Exercise 1(C)

1. 26766007 **2.** 53112709 **3.** 41300000 **4.** 1316063

5. (a) 10144124 (b) 18909559 **6.** 48622530

7. 11769245 **8.** 4356121 **9.** 8374134 **10.** 1782789

11. 8587868 **12.** 6392915 **13.** 1870770 **14.** ₹ 800625

15. 2395855 pens **16.** ₹ 1376100 **17.** 66300 km

18. 36872 **19.** ₹ 1895875 **20.** 17 kg 370 g **21.** 45 m

22. 1 m 85 cm **23.** 38 kg 500 g **24.** 1 kg 325 g **25.** 5 m

Exercise 1(D)

1. (a) 30 (b) 190 (c) 2780 (d) 27490 **2.** (a) 900 (b) 2200 (c) 54200 (d) 89400 **3.** (a) 1000 (b) 6000 (c) 28000 (d) 37000 **4.** (a) 30000 (b) 50000 (c) 20000 (d) 380000 **5.** (a) 70 (b) 140 (c) 70 (d) 810 (e) 640 (f) 640 **6.** (a) 600 (b) 900 (c) 800 (d) 6500 (e) 7700 (f) 39400

7. (a) 49000 (b) 58000 (c) 39000

8. (a) 60 (b) 30 (c) 260

9. (a) 400 (b) 500 (c) 2500

10. (a) 8000 (b) 8000 (c) 5000

Exercise 1(E)

1. (a) 2400 (b) 1600 (c) 2000 (d) 5400 (e) 2500 (f) 300 **2.** (a) 80000 (b) 40000 (c) 30000
3. (a) 28000 (b) 30000 (c) 20000 **4.** (a) 200000 (b) 40000 (c) 90000

Exercise 1(F)

1. 4	**2.** 4	**3.** 3	**4.** 35	**5.** 10
6. 10	**7.** 30	**8.** 20	**9.** 23	**10.** 23

Exercise 1(G)

1. (a) III (b) VII (c) XV (d) XXVIII (e) XXXIX (f) XLV (g) LVI (h) LXIII (i) LXXII (j) LXXX (k) XCII (*l*) XCVII (m) XCIX (n) CX (o) CXXV **2.** (a) CLIV (b) CLXXXV (c) CCXXX (d) CCCXLIII (e) CDLXXXV (f) DXCV (g) DCXIII (h) DCCLVII **3.** (a) 27 (b) 34 (c) 45 (d) 54 (e) 84 (f) 91 (g) 96 (h) 111 (i) 154 (j) 224 (k) 365 (*l*) 414 (m) 464 (n) 506 (o) 766 **4.** Do yourself

Exercise 1(H)

1. (b)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (b)
6. (b)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (a)

Exercise 2(A)

1. (a) 1, 2, 4, 5, 10, 20 (b) 1, 2, 3, 4, 6, 9, 12, 18, 36 (c) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 (d) 1, 3, 5, 15, 25, 75 **2.** (a) 17, 34, 51, 68, 85 (b) 23, 46, 69, 92, 115 (c) 65, 130, 195, 260, 325 (d) 70, 140, 210, 280, 350 **3.** (a) Even (b) Odd (c) Even (d) Even (e) Odd (f) Even (g) Odd (h) Even **5.** (a) 11, 13, 17, 19, 23, 29 (b) 37, 41, 43, 47, 53, 59 (c) 67, 71, 73, 79 (d) 83, 89, 97 **6.** (a) 2 (b) 2 (c) 3 **7.** (a) **8.** 90, 91, 92, 93, 94, 95, 96 **9.** Yes, 9 **10.** (59, 61), (71, 73) **11.** (2, 3), (3, 5), (6, 7), (4, 9), (8, 15); No **12.** (a) $36 = 7 + 29$ (b) $42 = 5 + 37$ (c) $84 = 17 + 67$ (d) $98 = 79 + 19$
13. (a) $31 = 5 + 7 + 19$ (b) $35 = 5 + 7 + 23$ (c) $49 = 3 + 5 + 41$ (d) $63 = 7 + 13 + 43$
14. (a) $36 = 17 + 19$ (b) $84 = 41 + 43$ (c) $120 = 59 + 61$ (d) $144 = 71 + 73$ **15.** None

Exercise 2(B)

1. (a), (b), (d), (e) **2.** (c), (f) **3.** (c), (d), (f) **4.** (a), (b), (d), (f) **5.** (a), (d) **6.** (b), (d), (e), (f) **7.** (d), (e), (f)
8. (b), (c), (e) **9.** (a), (c) **10.** (a), (c), (d), (e) **11.** (a) 2 (b) 1 (c) 0 (d) 2 (e) 1 (f) 2 **12.** (a) 2 (b) 3 (c) 7 (d) 8 (e) 4 (f) 6 **13.** 103, 137, 179 are prime numbers.
14. (a) 6 (b) 12 (c) 24 (d) 12 **15.** (a) F (b) T (c) F (d) T

Exercise 2(C)

1. $2^2 \times 7$ **2.** $2^3 \times 5$ **3.** 5×17 **4.** $2^5 \times 3$ **5.** $2^3 \times 3 \times 5$
6. $2^2 \times 5 \times 7$ **7.** 3×5^3 **8.** $2^5 \times 3 \times 5$ **9.** 5^4
10. $2^2 \times 5 \times 7^2$ **11.** 2^{10} **12.** 5^5 **13.** $3 \times 5 \times 17^2$
14. $3 \times 7 \times 13 \times 17$ **15.** $3^2 \times 17 \times 19$ **16.** $2^3 \times 3^2 \times 11^2$
17. $3^3 \times 7^2$ **18.** $11^3 \times 7$ **19.** $2^3 \times 3^2 \times 11^2$
20. $2^4 \times 3^2 \times 11^2$

Exercise 2(D)

1. (a) 5 (b) 8 (c) 6 (d) 2 (e) 33 (f) 5 (g) 9 (h) 7
2. (a) 20 (b) 28 (c) 12 (d) 6 (e) 7 (f) 5 (g) 36 (h) 12
4. 63 **5.** 87 **6.** 154 **7.** (a) $\frac{7}{9}$ (b) $\frac{8}{13}$ (c) $\frac{11}{17}$ **8.** 31*l*
9. 4290 tiles

Exercise 2(E)

1. (a) 60 (b) 90 (c) 28 (d) 420 (e) 360 (f) 300 (g) 25200 (h) 240 (i) 216 (j) 840 **2.** (a) 13; 1989 (b) 26; 5148 (c) 77; 9702 (d) 29; 1160 **4.** 1179
5. 435 **6.** 180 **7.** 8 **8.** 609 **9.** 99720 **10.** 10080
11. 225 cm **12.** After 3 hours **13.** 7 min 12 sec

Exercise 2(F)

1. (c)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (a)
6. (c)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (b)
11. (c)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (c)
16. (d)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (b)

Exercise 3(A)

1. 21000, 21001, 21002 **2.** 550000, 549999, 549998
3. 6506 **4.** 0 **5.** (a) 2100 (b) 32001 (c) 91470 (d) 504352 (e) 253525 (f) 100001 (g) 2482000 (h) 8989900 **6.** (a) 110 (b) 19899 (c) 20098 (d) 319999 (e) 91468 (f) 504350 (g) 235323 (h) 999999 **7.** 7510000, 7509999, 7509998
8. (a) F (b) T (c) F (d) T (e) F (f) T (g) F (h) T (i) F (j) F (k) F (*l*) T

Exercise 3(B)

1. (a) 263 (b) 468 (c) 489 (d) 6047 (e) 997
2. (a) 17000 (b) 3000 (c) 23400 **3.** 5522; 5522; Yes; Associative law of addition of whole numbers
4. (a) 2307 (b) 3200 (c) 25900 (d) 16800 (e) 200 (f) 400 **5.** (a) 110577 (b) 16783 **6.** Yes; by associative and commutative laws of addition of whole numbers
7. (a)

8	1	6
3	5	7
4	9	2

(b)

10	5	12
11	9	7
6	13	8

(c)

7	18	17	4
8	13	14	11
12	9	10	15
19	6	5	16

(d)

2	15	16	5
9	12	11	6
13	8	7	10
14	3	4	17

8. (a) T (b) F (c) T